# Energetic Analysis of Pentagon Road Intermediates of $\mathbf{C}_{60}$-Buckminsterfullerene Formation 

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#### Abstract

We report an energetic analysis of the principal intermediates of the pentagon road (PR) scheme for formation of $\mathrm{C}_{60}$-buckminsterfullerene. All calculations were initially performed using the tight-binding semiempirical method. For selected cases, more rigorous 3-21G/HF and 3-21G/B3LYP calculations were carried out. The first part of this study includes an energetic comparison between the $30-, 40$-, and 50 -atom PR intermediates and a representative group of $30-, 40$-, and 50 -atom carbon clusters. While $\mathrm{C}_{30}$ PR is higher in energy than a large variety of graphene sheets and fullerenes, $\mathrm{C}_{40}$ PR and $\mathrm{C}_{50}$ PR are considerably lower in energy than many other isomers; only fullerenes are more stable. Additionally, we examine a plausible mechanism by which $\mathrm{C}_{50}$ PR rearranges to form a $\mathrm{C}_{50}$ cage with $D_{5 h}$ symmetry. Because of its large energy barrier, this process is unlikely to affect the $\mathrm{C}_{60}$ growth mechanism.


## Introduction

$\mathrm{C}_{60}$-buckminsterfullerene (BF) was first detected as a product arising from laser-vaporized graphite. ${ }^{1}$ Mass spectrometry and the recently developed technique of ion chromatography ${ }^{2}$ give a large amount of information on carbon species present following graphite vaporization. Many schemes of fullerene formation draw from this growing source of data. ${ }^{3-8}$ A wellknown mechanism for the formation of $\mathrm{C}_{60} \mathrm{BF}$, dubbed the pentagon road (PR), ${ }^{4}$ explains fullerene formation by imposing the constraint that developing clusters minimize their number of dangling bonds by incorporating a maximal number of nonabutting pentagonal rings. Some of the key intermediates of the PR mechanism are shown in Figure 1A. For the purpose of this work, we assume that $\mathrm{C}_{20}$ PR goes to $\mathrm{C}_{60} \mathrm{BF}$ by stepwise addition of $\mathrm{C}_{2}$ fragments along the rim of the growing carbon cluster. The portion of this mechanism going from $\mathrm{C}_{20} \mathrm{PR}$ to $\mathrm{C}_{30} \mathrm{PR}$ is outlined in Figure 1B. Previous theoretical studies were carried out to determine the energetics underlying $\mathrm{C}_{2}$ addition and other types of annealing and fragmentation processes for various fullerene systems. ${ }^{9,10}$

The PR mechanism contains several advantageous features. First, many atoms on the cluster rim make up armchair edges, thus minimizing the number of dangling bonds. Furthermore, these intermediates avoid some bonding patterns that are energetically unfavorable, such as adjacent five-membered carbon rings. Issues concerning energetic penalties arising from dangling bonds and curvature strain for similar carbon systems have recently been discussed elsewhere. ${ }^{11}$

Theoretical analysis may reveal important information about the stability of individual PR structures and the overall feasibility of the mechanism. In this study, we perform tight-binding calculations followed by high-quality $a b$ initio calculations to determine the energetics of important intermediates predicted by the PR scheme of fullerene growth. In addition, we investigate a mechanism going from $\mathrm{C}_{50} \mathrm{PR}$ to the $\mathrm{C}_{50}$ cage with $D_{5 h}$ symmetry. This mechanism may represent an important divergent path from the PR.

## Computational Details

Calculations were initially performed using the semiempirical tight-binding (TB) potential for carbon. ${ }^{12}$ Properties of

[^0]
## A


$\mathrm{C}_{20} \mathrm{PR}$

$\mathrm{C}_{30} \mathrm{PR}$


B










Figure 1. Structures of pentagon road intermediates.
the tight-binding potential and functional form are discussed thoroughly elsewhere. ${ }^{12,13}$ These calculations were compared with more rigorous calculations using the 3-21G basis set with Hartee-Fock (HF) and Becke's three-parameter hybrid method using the Lee, Yang, and Parr correlational functional (B3LYP). ${ }^{14,15}$ All calculations were carried out using a development version of the GAUSSIAN suite of programs. ${ }^{16}$ All TB energy values quoted in this paper correspond to molecules with geometries fully optimized by a TB conjugate-gradient scheme. HF and B3LYP energy values were obtained at molecular geometries fully optimized at the $3-21 \mathrm{G} / \mathrm{HF}$ level of theory.

## Results

To establish an overall picture of the energetics of the PR mechanism, we carried out TB and 3-21G/HF calculations on


Figure 2. TB (dashed line) and HF (solid line) energies of the reactions $\mathrm{C}_{60-2 m} \mathrm{PR}+m * \mathrm{C}_{2}(m=0-20)$, plotted as a function of the number of atoms in each molecule.

TABLE 1: Comparison of the Energies (eV) of Various 30-Atom Carbon Isomers Relative to $\mathrm{C}_{30}$ PR (Entry A); See Also Figure 3

| molecule | tight-binding $^{a}$ | $3-21 \mathrm{G} / \mathrm{HF}^{b}$ | 3-21G/B3LYP |
| :---: | :---: | :---: | :---: |
| A | 0.0 | 0.0 | 0.0 |
| B | -8.9 | -11.3 | -11.3 |
| C | -7.1 | -8.3 | -8.9 |
| D | -5.4 | -7.3 | -6.5 |
| E | -4.4 | -6.5 | -5.7 |
| F | -4.7 | -4.4 | -3.5 |
| G | -3.6 | -2.1 | -1.1 |
| H | -2.0 | -0.5 | 1.4 |
| I | -2.0 | 0.4 | 1.9 |
| J | -3.4 |  |  |
| K | -2.0 |  |  |
| L | 0.5 |  |  |
| M | 0.9 |  |  |
| N | 2.4 |  |  |
| O | 10.6 |  |  |
| T |  |  |  |

${ }^{a}$ Tight-binding optimized geometry. ${ }^{b} 3-21 \mathrm{G} / \mathrm{HF}$ optimized geometry.


A


B


C


G


D

H


I


J


K

L


M


N


0

Figure 3. Structures referred to in "molecule" entry of Table 1.
intermediates from $\mathrm{C}_{20} \mathrm{PR}$ to $\mathrm{C}_{60} \mathrm{BF}$. In Figure 2, the energies of the reactions $\mathrm{C}_{60-2 m} \mathrm{PR}+m \mathrm{C}_{2}(m=0-20)$ are plotted as a function of the number of atoms in each PR cluster. The

TABLE 2: Comparison of the Energies (eV) of Various 40-Atom Carbon Isomers Relative to $\mathrm{C}_{40}$ PR (A); See Also Figure 4

| molecule | tight-binding $^{a}$ | $3-21 \mathrm{G} / \mathrm{HF}^{b}$ | 3-21G/B3LYP |
| :---: | :---: | :---: | :---: |
| A | 0.0 | 0.0 | 0.0 |
| B | -1.9 | -1.2 | -1.9 |
| C | -0.8 | 0.4 | -0.5 |
| D | 1.2 | 1.8 | 1.6 |
| E | 0.3 | 2.6 | 1.3 |
| F | 3.6 | 3.0 | 3.9 |
| G | 3.3 | 8.1 | 8.4 |
| H | 4.1 |  |  |
| I | 6.3 |  |  |
| J | 9.2 |  |  |
| K | 9.6 |  |  |
| L | 11.6 |  |  |
| M | 11.7 |  |  |
| N | 12.0 |  |  |
| O | 12.3 |  |  |
| P | 12.4 |  |  |
| Q | 13.7 |  |  |
| R | 15.7 |  |  |

${ }^{a}$ Tight-binding optimized geometry. ${ }^{b}$ 3-21G/HF optimized geometry.


A


B


C


D


E


F


G


H


J


K


L

M

N


0

P


Q


R

Figure 4. Structures referred to in "molecule" entry of Table 2.
steady decrease in energy with growing size of the PR intermediates clearly shows that the PR mechanism is energetically favorable as a whole. However, a signifigant energy barrier is seen for the reaction of $\mathrm{C}_{50} \mathrm{PR}$ and $\mathrm{C}_{2}$ to yield $\mathrm{C}_{52} \mathrm{PR}$. We predict the magnitude of this barrier to be greater than 1.8 and 3.0 eV , using TB and $3-21 \mathrm{G} / \mathrm{HF}$ methods, respectively. The signifigance of this energy barrier will be discussed in more detail below.

Next, we examined the stability of the $30-$, $40-$, and 50 -atom PR intermediates as constrasted with other isomers. Results obtained for carbon chains, rings, and cycloadducts ${ }^{7}$ are

TABLE 3: Comparison of the Energies (eV) of Various 50-Atom Carbon Isomers Relative to $\mathrm{C}_{50}$ PR (A); See Also Figure 5

| molecule | tight-binding $^{a}$ | $3-21 \mathrm{G} / \mathrm{HF}^{b}$ | 3-21G/B3LYP ${ }^{b}$ |
| :---: | :---: | :---: | :---: |
| A | 0.0 | 0.0 | 0.0 |
| B | -8.5 | -8.2 | -7.7 |
| C | 7.3 | 14.2 | 14.6 |
| D | 7.1 | 14.6 | 14.9 |
| E | 8.6 | 16.9 | 18.0 |
| F | 8.6 |  |  |
| G | 9.2 |  |  |
| H | 9.3 |  |  |
| I | 10.3 |  |  |
| J | 14.4 |  |  |
| K | 14.5 |  |  |
| L | 18.0 |  |  |
| M | 18.3 |  |  |
| N | 21.9 |  |  |
| P | 23.4 |  |  |

${ }^{a}$ Tight-binding optimized geometry. ${ }^{b} 3-21 \mathrm{G} / \mathrm{HF}$ optimized geometry.

A
A


E


I


M


B


F


J


K


L


N


O


P

Figure 5. Structures referred to in "molecule" entry of Table 3.
excluded because they are energetically noncompetitive with the PR intermediates. For $\mathrm{C}_{30}$, the monocyclic ring, linear chain, and $4+4$ cycloadduct are higher in energy than $C_{30} \mathrm{PR}$ as determined by TB calculations by $2.4,4.7$, and 3.3 eV , respectively. Similar results hold for $\mathrm{C}_{40}$ and $\mathrm{C}_{50}$. As shown in Table 1, $\mathrm{C}_{30}$ PR (Figure 3, entry A, denoted 3A) is higher in energy than a wide variety of molecules in our 30 -atom comparison set ( $3 \mathrm{~B}-3 \mathrm{O}$ ). Two 30 -atom fullerenes ( $3 \mathrm{~A}, 3 \mathrm{~B}$ ) are clearly lower in energy than any of the other 30 -atom clusters examined in this study. $\mathrm{C}_{30}$ cage with $C_{2 \mathrm{v}}$ symmetry ( 3 B ) is lower in energy than $\mathrm{C}_{30}$ PR by $8.9,11.3$, and 11.3 eV , using TB, 3-21G/HF, and 3-21G/B3LYP, respectively. These results are consistent with those found for isomers of $\mathrm{C}_{24} .{ }^{17}$

In contrast to $\mathrm{C}_{30} \mathrm{PR}, \mathrm{C}_{40} \mathrm{PR}$ (Figure 4 A ) is much more competitive in energy with molecules in our 40 -atom comparison set (4B-4R). As shown in Table 2, $\mathrm{C}_{40} \mathrm{PR}$ is lower in energy than all tested graphene sheets and fullerenes except the $\mathrm{C}_{40}$ fullerene with $C_{2}$ symmetry (4B). This fullerene is lower


Figure 6. Graph depicting the potential energy surface of one mechanism from $\mathrm{C}_{50} \mathrm{PR}$ to a $\mathrm{C}_{50}$ cage with $D_{5 h}$ symmetry. Energies were calculated using TB (dashed) and $3-21 \mathrm{G} / \mathrm{HF}$ (solid) methods.


Figure 7. Structures referred to in Figure 6.
in energy than $\mathrm{C}_{30}$ PR by $1.9,1.9$, and 1.2 eV , using TB, 3-21G/ HF, and $3-21 \mathrm{G} / \mathrm{B} 3 \mathrm{LYP}$, respectively.

Results for $\mathrm{C}_{50} \mathrm{PR}$ (5A) were similar to those obtained for $\mathrm{C}_{40} \mathrm{PR}$. As seen in Table 3, the energy difference between the PR structures and chosen graphene sheets continues to increase. Species in Table 3 are depicted in Figure 5. $\mathrm{C}_{50}$ with $D_{5 h}$ symmetry (5B) is lower in energy than the PR structure by 8.5 , 8.2 , and 7.7 eV using TB, 3-21G/HF, and 3-21G/B3LYP, respectively.

As shown in Figure 2, the energy of the reaction of $\mathrm{C}_{50}$ and $\mathrm{C}_{2}$ to make $\mathrm{C}_{52}$ is energetically uphill. Furthermore, as shown in Table 3, the $\mathrm{C}_{50}$ cage with $D_{5 h}$ symmetry is considerably lower in energy than $\mathrm{C}_{50} \mathrm{PR}$. These results demand further


Figure 8. Graph depicting the potential energy surface of a second mechanism from $\mathrm{C}_{50} \mathrm{PR}$ to a $\mathrm{C}_{50}$ cage with $D_{5 h}$ symmetry. Energies were calculated using TB (dashed) and 3-21G/HF (solid) methods.


Figure 9. Structures referred to in Figure 8.
investigation of paths from $\mathrm{C}_{50} \mathrm{PR}$ that diverge from the PR mechanism. One important example of such a pathway is the stepwise conversion of $\mathrm{C}_{50} \mathrm{PR}$ to $\mathrm{C}_{50}$ with $D_{5 h}$ symmetry. In Figure 6, we show the potential energy surface of one plausible mechanism for this reaction. Each point on the graph refers to a structure depicted in Figure 7. Mechanism 1 begins by consecutive 1,2-rearrangements of two adjacent armchair edges of $\mathrm{C}_{50} \mathrm{PR}(7 \mathrm{~A})$ to produce two $\mathrm{C}_{2}$ sticks (7B, 7C). These two $\mathrm{C}_{2}$ sticks bond to form a rim to which additional adjacent $\mathrm{C}_{2}$ sticks may bond (7D). In the following steps, additional 1,2rearrangements on adjacent armchair edges result in completion of the rim and closure of $\mathrm{C}_{50} \mathrm{PR}$ into a $\mathrm{C}_{50}$ cage with $D_{5 h}$ symmetry. The energy barrier of this mechanism is larger than
the highest energy of any one of the intermediates relative to $\mathrm{C}_{50} \mathrm{PR}$, which is 4.5 and 3.7 eV using TB and $3-21 \mathrm{G} / \mathrm{HF}$, respectively.

Energies for an additional mechanism for the above reaction are shown in Figure 8. Each point on the graph refers to a structure depicted in Figure 9. This mechanism is similar to that for Figure 6, except the first two 1,2-rearrangements occur opposite one another (9B, 9C). This requires that a third 1,2rearrangement occur (9D) before a rim composed of three bound sticky ends can form (9E). This mechanism then proceeds identically to mechanism 1 , resulting in a $\mathrm{C}_{50}$ cage with $D_{5 h}$ symmetry (9I). The energy barrier of this mechanism is 5.7 and 5.6 eV according to our TB and $3-21 \mathrm{G} / \mathrm{HF}$ predictions, respectively.

The results of our study provide an energetic evaluation of important intermediates of the pentagon road mechanism for $\mathrm{C}_{60} \mathrm{BF}$ formation. In contrast with $\mathrm{C}_{30} \mathrm{PR}$, which is higher in energy than most 30 -atom graphene sheets in our comparison set, $\mathrm{C}_{40}$ and $\mathrm{C}_{50} \mathrm{PR}$ are energetically competitive with a variety of $40-$ and 50 -atom graphene sheets. However, $30-$, $40-$, and 50 -atom fullerenes are lower in energy than each of these PR intermediates. Also, an energy barrier between $\mathrm{C}_{50}$ and $\mathrm{C}_{52} \mathrm{PR}$ suggests the possible existence of alternative mechanisms from $\mathrm{C}_{50}$ PR. We examined two such paths from $\mathrm{C}_{50}$ PR to a lower energy $\mathrm{C}_{50}$ isomer, $\mathrm{C}_{50}$ with $D_{5 h}$ symmetry. However, we predict that energy barriers for these two mechanisms are too high to be reached at experimental temperatures for high-yield synthesis of $\mathrm{C}_{60}$.

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